# Design of Stable Recurrent Fuzzy-logic-controllers

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## ABSTRACT

Recently, there is great deal of interest in the use of fuzzy expert systems in control applications. Controllers based on fuzzy logic belong to the class of static or memoryless nonlinear controllers and provide better control than is possible using linear control. The major strength of fuzzy controllers lies in the way a nonlinear output mapping of a number of inputs can be specified easily using fuzzy linguistic variables and fuzzy rules.

When we use fuzzy logic in feedback control, we in effect have a nonlinear system with feedback and the resulting system could potentially become unstable. In this work, we show how we can design feedback systems (with fuzzy control) that are guaranteed to be stable. This approach can also be used to design stand-alone feedback fuzzy systems (or recurrent fuzzy systems) that are guaranteed to be stable. Due to space limitations, we provide only one simple example here, but will provide more details including the use of our new approach for the famous truck-backer-upper problem at the conference.

## INTRODUCTION

Linear control is a most commonly used method with a variety of techniques and has been successfully applied in a number of industrial applications. The reasons for their continued acceptance/use are that linear systems and linear controls are easier to characterize analytically and issues such as stability, performance and robustness can be analyzed and quantified accurately. However, linear controls have a number of limitations. First they rely on the linear model assumptions being valid. When this assumption is violated, the system performance can degrade and the system can even become unstable.

Linear control is the product of the technologies (or lack of it) of some two decades ago. Since then, the technology has changed rapidly and the complexity of systems to be controlled have increased dramatically. Thus, the need for nonlinear controllers have increased along with the feasibility for their cost-effective implementation.

Controllers based on fuzzy expert systems can be considered as a class of static (memoryless) nonlinear controllers [1,2] In the case of fuzzy controllers, the input(s) to output(s) mapping (in general nonlinear) is achieved through the following three steps:

a) fuzzification or input fuzzy set(s) activation from the crisp input sensor(s) values;

b) output fuzzy set(s) selection from the input fuzzy sets activated and the fuzzy rule base;

c) defuzzification or crisp output(s) values calculation based on the output fuzzy sets selected, the membership function values corresponding to the input(s) values and the membership functions corresponding to the output fuzzy sets selected.

The importance of fuzzy logic for control applications arise from the fact that a proper nonlinear mapping that would lead to a superior controller performance can be described easily using fuzzy linguistic variables and fuzzy rules.

Feedback architecture is the most preferred architecture (Figure 1) whether we use linear or nonlinear/fuzzy controllers due to reasons such as reduced sensitivity to plant coefficient variations and so on. The stability becomes an important issue in such feedback architectures. There is lot of research/literature on the stability of linear feedback systems. However, there are no known results that establish mathematically the stability of nonlinear fuzzy controller based feedback

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systems. Hence, there is hesitancy on the use of nonlinear/fuzzy controllers in applications where human safety etc. are involved. In this paper, we arrive at a new feedback architecture using fuzzy controllers. We will show that such an architecture will remain stable regardless of the fuzzy controller mapping used. This method can also be used to design stand-alone feedback fuzzy systems or recurrent fuzzy systems that are guaranteed to be stable.

## THEORY

An impressive number of tools for the analysis of nonlinear systems and techniques for controller design for nonlinear systems have been developed [3-6]. However, all of these techniques are analytically motivated (defining general form of nonlinear dynamical differential equations for representing systems and placing restrictions to satisfy stability etc.). However since nonlinear differential equations are highly complicated to be amenable for analytical approach, restrictions have been placed on the types of systems that can be analyzed and the types of controllers that can be employed etc.

Our approach for nonlinear system/controller design (that is equally applicable for design of feedback systems with fuzzy controllers and the thrust of this paper) is based on an entirely different paradigm. The proposed approach is based on *"Engineering"* and *"Reverse-Engineering"* philosophies as opposed to analytical/mathematical points of view adapted by earlier researchers. An engineering or physically motivated approach tries to take into consideration physical properties and constraints-based-on-physical-properties at every stage of design. Passivity formulation (to be defined shortly) is one such candidate that we use in our work. The term "Reverse-Engineering" is used here to imply learning from an existing system (not necessarily of the same kind) and plays an important role in our work. Thus, to design a nonlinear fuzzy controller, we force the closed-loop system dynamics to mimic[1] the dynamics of another system that is guaranteed to be stable, a process or technique similar to reverse engineering. Assuming such a system exists, making the closed-loop system mimic that system is indeed possible since we have the flexibility in the choice of the controller. We have used *passivity* concepts[2] to arrive at such a stable system, *a passive nonlinear electrical network.* The passivity concepts are used to first invent two new passive nonlinear electrical devices[3]: nonlinear transformers (2-port device) and multi-port gyrators. A proper interconnection of such elements with dynamical elements (capacitors and inductors) provides the target stable system. The controller dynamics are then chosen such that the closed-loop system dynamics mimic the dynamics of that stable electrical network.

Let us now illustrate the application of this approach to the design of fuzzy controllers using Figure 2. In Figure 2A, we show the classical architecture whose stability is open to question. In Figure 2B, we show the general architecture based on the passive nonlinear network approach and a nonlinear controller with a first-order dynamics[4]. Assuming that the plant can be represented by a 2nd-order transfer function with two state variables $y, \dot{y} = y_1$, the dynamics of the close-loop system can be written as

$$\begin{bmatrix} \dot{y} \\ \dot{y}_1 \\ \dot{k} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -a & -b & F(e,\dot{e}) \\ 0 & \frac{-F(e,\dot{e})}{a} & 0 \end{bmatrix} \begin{bmatrix} y \\ y_1 \\ k \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ f(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix}$$

where the dynamics of the controller are chosen so as to make the close-loop dynamics (third-order) mimic a nonlinear network with 3 dynamic elements, and the new device called nonlinear gyrator (a multi-port, lossless, and memoryless device) as shown in Figure 3. The mapping, f(k) has to be a passive (lossy) resistive mapping. That is, {k, f(k)} should be confined to the first and third quadrants. The input $u_1$ will be chosen so as to force k(t) to a particular constant as $e, \dot{e} \rightarrow 0$.

The above in a nut shell describes the methodology that would lead to the design of nonlinear fuzzy controllers that are guaranteed to be stable. Though the concept may sound

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[1] The term "mimic" is used here to imply exactly the same performance. Thus, if the two systems were treated as two black boxes, we would not be able to identify them from external measurements.

[2] In this section, we provide only salient points of out approach. A complete description is provided in appendix A.

[3] We plan to obtain patent protection on these devices.

[4] The new architecture will be patented.

simple, it is a very powerful methodology and can be applied to a number of applications other than fuzzy controller design and would silence critics who tend to raise questions such as "would you travel in an aircraft controlled by a fuzzy controller whose stability properties are unknown".

## SIMULATIONS

To illustrate this concept, we have taken a third order model example used in reference [7], retained only the two dominant poles and used the fuzzy look-up table given in that paper with some modifications to generate the fuzzy controller output $F(e,\dot{e})$. Denoting the transfer function of the plant as

$$H(s) = \frac{b}{s^2 + as + b} = \frac{Y(s)}{U(s)}$$

with u as input to the plant, and y the output of the plant, the dynamics of the complete system is given by

$$\dot{y} = y_1$$

$$\dot{y}_1 = -by - ay_1 + u$$

$$u = kF(e,\dot{e})$$

$$\dot{k} = -y_1F(e,\dot{e}) - k - \frac{4}{\pi}\tan^{-1}(k) + u_1$$

where $u_1$ is chosen to force k to a particular value as the plant output moves to the target value. The terms $k$ and $\frac{4}{\pi}\tan^{-1}(k)$ have been chosen based on the network requirements and correspond to the currents in two parallel resistors, one being nonlinear and the other linear. The form and the exact values were chosen rather arbitrarily for this example and the proper choice would lead to the optimal performance. Though the simulation results could not be shown here due to space limitations, he responses of the plant using the new network based approach for two values of $k(\infty$ ) showed some improvement in the response as compared to the response with the classical fuzzy controller. However, the key point here is that the system represented by the above set of equations will remain stable and robust for external disturbances.

## SUMMARY

We have proposed a new method that guarantees the stability of feedback systems with fuzzy controllers. Instead of using a fuzzy expert system as a simple static (no memory) nonlinear mapping device, the new approach arrives at a fuzzy controller with memory (Figure 2B). This approach is also suitable for stand-alone fuzzy expert systems with internal feedback.

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## APPENDIX Stable Nonlinear System Design

Passivity is a term commonly used in Electrical Network Theory to indicate consumption of power and energy. A passive electrical element (linear or nonlinear) is one which always consumes power (lossy) or at the most, consumes no power/energy (lossless). They can be non-dynamic (no memory or cannot store energy) or dynamic (stores energy and gives it back at some other time). They can be two-terminal (one-port) elements or multi-terminal (multi-port) devices. A passive linear or nonlinear network is simply an electrical network formed by proper interconnection of various passive linear or nonlinear elements. The interconnections must be such that the basic circuit laws are obeyed. An important property of such networks is that they are globally asymptotically stable (we can also achieve BIBO stability by putting some additional restrictions) and will remain so as long as the characteristics of the individual elements remain in the permissive range for passivity. Thus, if we have proper passive nonlinear elements, we can form stable nonlinear networks, obtain dynamical equations describing such networks in terms of the element characteristics and use them as target equations for the nonlinear system to be designed.

The above approach assumes that proper nonlinear passive elements exist. Some nonlinear electrical elements have been identified in the open literature. One such element is a nonlinear resistor. An example of a passive nonlinear resistor is one with a current-voltage relationship given by by the sigmoidal function as: $i_R(t) = G \tan^{-1}(v_R(t)); \quad G > 0$. The power p(t) consumed by this element is: $p(t) = i_R(t)\, v_R(t)$ and is always non-negative. In general, the v-i characteristic of a general nonlinear passive resistor has to be confined to the first and third quadrants in the v-i plane and has to pass through the origin. It should be noted that a general nonlinear resistor: 1) is defined to be either voltage controlled or current controlled, implying that caution should be exercised in forming networks with them (and other nonlinear elements), 2) has an element characteristic that need not be bilateral (for bilateral property), and 3) can have a small-signal resistance (or conductance) which can be positive, zero or negative with out violating the passivity constraint. Whether we need to have all these general properties can be decided from a practical perspective. For example, a negative small-scale resistance simply introduces a 180 degree phase shift (when one increases, the other decreases and vice-versa) and may not be really necessary in most applications.

Nonlinear capacitors and inductors (lossless dynamical elements) have also been defined. A nonlinear capacitor can be either charge controlled or voltage controlled. For a charge controlled capacitor we will have:

$$v_c = v_c(q_c), \quad i_c = \frac{dq_c}{dt} \quad \text{and}$$

$$\dot{v}_c = \frac{dv_c}{dt} = \frac{dv_c}{dq_c}\frac{dq_c}{dt} = S(q_c)\, i_c$$

where $q_c$ is the charge, $v_c$ the voltage and $i_c$ the current at any time, and $S(q_c)$ is the reciprocal small-signal capacitance (function of $q_c$ and can be positive or negative), and is independent of time (nonlinear time-invariant capacitor). It can be shown that a nonlinear time-invariant (NTI) capacitor (as well as an inductor) is also lossless (neither dissipates nor generates power) if the charge Vs. voltage (flux Vs. current) characteristic is confined to the first and third quadrants and pass through the origin[5]. Thus the power entering the device is stored as energy during certain time and returned back at other time.

A nonlinear time-invariant capacitor can be voltage controlled, leading to small-signal capacitance $c(v_c)$ definition. However, this form would require the inversion of $c(v_c)$ if the network equations are arranged in the state-space form (involving $v_c$ ) and solved numerically for a solution. Thus, we would restrict to charge-controlled capacitors ($q_c$ would then be the state-variable) and similarly to flux-controlled inductors (flux becomes the state variable). We omit the expressions for inductance due to space limitations.

In addition to these three known nonlinear elements, we have defined/invented a number of other passive nonlinear devices. One such device is a nonlinear transformer (A patent application has been filed for the new devices introduced in this proposal and many of their uses), a two-port element with the transfer characteristics given by:

$$\begin{bmatrix} v_2(t) \\ i_1(t) \end{bmatrix} = \begin{bmatrix} N(\cdot) & 0 \\ 0 & -N(\cdot) \end{bmatrix} \begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}$$

where $N(\cdot)$ is a nonlinear function of the currents(s) and voltage(s) in an electrical network in which the transformer is embedded. It should be noted that $v_1(t)\, i_1(t) + v_2(t)\, i_2(t) = 0$ for all t regardless of what ever form $N(\cdot)$ takes. Thus, an ideal nonlinear transformer is a lossless, non-dynamic (or memoryless) two-port device. Both an analog and digital (hardware or micro-computer based) implementation of this device (and other devices invented) is possible. The digital implementation would allow us to realize

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[5] Lossless nonlinear reactive elements with characteristics on the second and fourth quadrants are also possible. However, such elements will lead to complex behaviors, such as multiple equilibrium points, and hence care must be exercised in their use.

such elements/devices with ideal characteristic as defined with no implementation errors even under finite-precision arithmetic.

An example of a nonlinear transformer would be one with a transformer ratio N( ) given by:

$$N(e) = 1 \ \text{if} \ |e| \geq 1; \ |e| \ \text{otherwise}$$

where e is some error function that we expect to go down or minimize. This transformer could be used in conjunction with a fixed resistor (linear or nonlinear) terminated at the 2nd port. The load seen at port 1 would be a time-varying resistance which becomes infinite (open circuit) as the error e -> 0. Thus, we will have a damper in the circuit which gets removed as the equilibrium is reached. We have used such a combination in adaptive control described later.

Another device that we have invented and that is highly useful is that of a multi-port nonlinear gyrator.If we denote $V = [v_1, v_2, ..., v_N]^T$, $I = [i_1, i_2, ..., i_N]^T$, where $v_1, v_2, ..., v_N$ are the voltages across the gyrator ports, and $i_1, i_2, ..., i_N$ the currents into the ports of a N-port gyrator and T for transpose, we have the relationship between the voltages and currents as follows: I = Y V, where $Y = [yij(.)]$, $i, j = 1 \text{ to } N$ is the admittance matrix of the nonlinear gyrator and satisfies the constraint $Y + Y^T \equiv 0$. The elements $yij(.)$ can be complex functions of the current(s) and voltage(s) (in general, the state variables) in an electrical network. We can show that for a general N-port gyrator:, $[I]^T [V] \equiv 0$, for any functions $yij(.)$. That is, a nonlinear gyrator is a lossless and non-dynamic multi-port device.

We can interconnect these nonlinear, passive elements to arrive at different networks and write their corresponding nonlinear, dynamical equations in the form of state-space equations. The number of dynamical elements in the network would define the number of state-variables or number of state-space equations and other elements (as well as the dynamical elements) would define the nonlinear characteristics of the dynamics and the degrees of freedom. Thus, the prototype electrical network that we form would depend on the system that needs to be modeled and or controlled.

Higher-order (say, N-th order) networks and their corresponding nonlinear, dynamical equations can be obtained by terminating a N-port gyrator with capacitor/transformer and resistor/transformer combinations and independent current sources. Assuming charge controlled nonlinear capacitors, the nonlinear dynamic equations in the state-space form for the network can be written using the Kirchoff's current law as: $\dot{Q} = Y \, V(Q) - F(V(Q)) + I_s$ where Q is a vector of capacitor charges (state-vector) and of size N, V is the vector of voltages, $\dot{Q}$ is the vector of derivatives of charges, F(V) the vector of currents through the nonlinear resistors, Y the admittance matrix of a N-port nonlinear gyrator (or the admittance matrix of a N-port lossy gyrator formed by terminating the last M ports of a lossless N+M port nonlinear gyrator with nonlinear passive resistors), and $I_s$ vector of source currents. It can be observed (and proven easily using Lyapunov theorem for stability) that the set of equations taken from a passive network represents a globally stable network or system as long as the element values are in the permissible range so as to retain the lossy or lossless property. The stability property holds good even if the terms in the various nonlinear elements are defined by complex functions and the network is highly complex. If such a system is turned on with only initial stored energies in the dynamic elements, all the state variables will go to zero as time progresses.

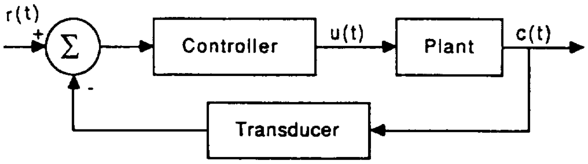
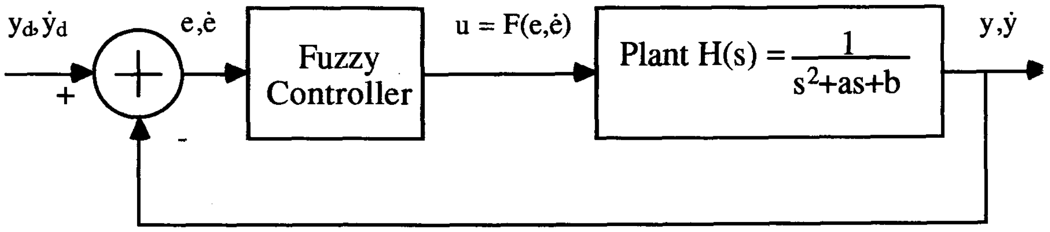
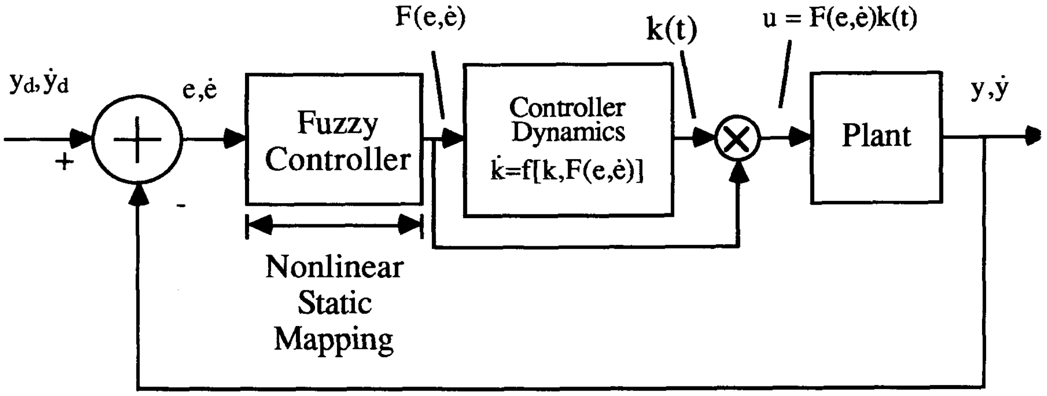


**Figure 1.** General feedback control system architecture.



A. Classical Fuzzy Control Approach



B. New Nonlinear Network Based Approach

**Figure 2. A. Classical approach. B. New Network based Architecture**

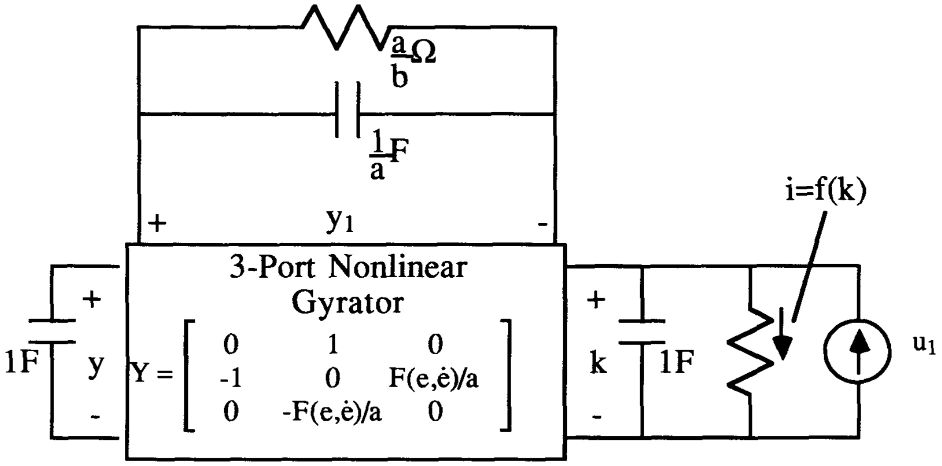


**Figure 3. Network Equivalent of Fuzzy Controller**