NONLINEAR SIGNAL PROCESSOR DESIGN: A BUILDING BLOCK APPROACH

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ABSTRACT

A new approach for the design of nonlinear systems (signal processors) based on an entirely new and interesting paradigm is presented in this paper. It is similar in spirits to the Neural nets paradigm (learn from existing systems) or the object oriented approach that has now become standard in software engineering and, in fact, takes it one step further as explained in the paper. This approach, though very simple, makes complex nonlinear systems design much easier. A number of practical applications are possible and we discuss few such applications. Detailed results will be provided at the conference

1. INTRODUCTION

Nonlinear dynamics is a highly complex topic and appears in many applications such as control; signal processing, neural net, chaos etc. Even among these applications, researchers prefer to specialize in very specific problems, such as median filtering, third-order dynamics that lead to chaos and so on, because of the complexity and the resulting richness in behavior of such circuits.

Most prior research in application of nonlinear dynamics uses an *analytical/mathematical approach.* It is sort of a *top-down approach* where one starts with the general form of differential equations (nonlinear in this case), incorporate constraints to arrive at special classes of differential equations, impose further conditions such as stability and use certain classes for certain application domains. Naturally, as we move down, the domain for such entities shrinks or reduces. More importantly, any physical insight that might otherwise be available about the system is rarely used in this approach. In this paper, we introduce a new approach for the design of nonlinear and adaptive/learning dynamical systems that may be called as a *"Building block approach"* or *"Engineering and Reverse-Engineering approach,"* or "Synthesis by Analysis," or *"Bottom-up approach"*, and is explained below. We will also discuss the application of the new paradigm to a number of application areas.

2. DESIGN PHILOSOPHY

A new approach for the design of nonlinear signal processors (and other nonlinear systems) is presented in this paper. It is similar in spirits to the Neural nets paradigm (learn from existing systems) and, in fact, takes it one step further. Conceptually, we start with an "*element box*" (similar to a toolbox) containing single-port and multi-port elements or *building blocks* necessary to build complex nonlinear and or time-varying electrical networks and circuits. These elements, most of them well known [1-4][1], are characterized by mathematical relationships connecting their input and output variables[2]. We can make them 1) completely lossy (consume power all the time) or 2) static-lossless, or 3) dynamic-lossless (able to store energy, return it later, and has a well defined relaxation point or points), or 4) lossy or generates power depending on the value of the independent variables and or 5) completely active by defining their I/O characteristics properly. We need to observe certain interconnection-rules (ex: avoiding inductors and current source nodes and capacitors and voltage source loops) when interconnecting such elements. Thus complex, nonlinear electrical circuits can be formed from such elements subject to the interconnection constraints. In general the resulting

[1] The last two papers describe, through a number of simple examples, how a straight-forward extension of some of the LTI system theory concepts of Losslessness and passivity to nonlinear systems can lead to some peculiar problems. However, all these problems can be taken care of if we make the assumptions necessary in practical life. Reactive elements will have at least one finite-valued (not necessarily zero) relaxation point where the available energy equals zero. The magnitude of the dependent variables becomes infinite as the magnitude of the independent variables becomes infinite. Since our goal here is the design of nonlinear systems, and in any design we go for conservative techniques, such assumptions will not have any negative impact on the proposed design paradigm.

[2] In a digital realization, we are more interested in these I/O relationships, and not the actual analog elements.

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circuit will have five well-defined sub-networks (each containing one kind of element as discussed before) all of which may or may not be present depending upon the application.

From the properly formed nonlinear electrical circuit, we can obtain a general description of a nonlinear dynamical system in the form of a state-space representation as:

$$\dot{\mathbf{x}} = \mathbf{f}[\mathbf{g}[\mathbf{x}]\mathbf{u}] \tag{1}$$

where $\mathbf{x}$ is the state vector of size N, $\mathbf{g}[\mathbf{x}]$ is the vector of nonlinear functions corresponding to the individual element I/O relationships, u is the vector of inputs, and $\mathbf{f}$ is a vector of nonlinear functions arising from the interconnections and so on. A family of state equations with the desired global properties can be obtained from the general form in (1) by varying $\mathbf{g}[\mathbf{x}]$ within their permissible sets. For example, to obtain dynamics with the origin as a locally, asymptotically stable equilibrium point, we will use lossless, dynamic elements with the origin as their relaxation points and lossy elements, and avoid active elements. The dynamics can be made globally, asymptotically stable if the lossless dynamic elements have no other relaxation points and have stored energy characteristics that are monotonically increasing[3]. Of course, if the application demands multiple equilibrium points, we can take care of them through the use of reactive elements with multiple equilibrium points or non-passive elements. Further, by placing restrictions on how and where the various sources will be connected we can ensure the *total stability* of the dynamics.

To design a new system (electrical or non-electrical), we can use the state-space representation obtained from the passive circuit as a guide or a template. Readers may notice that this is similar to "*Reverse Engineering*" where one builds a new system as quickly as possible by studying the inner workings of an existing system (perhaps by a competitor), and copying as much as possible (without putting themselves in the position of breaking the patent laws!). Here we take this art one step further in that the "system to be copied" exists and or formed in an *abstract* form, the *generic* form of the nonlinear dynamics are derived from such systems, and adapted to match the environment (of the specific design problem). Of course, no patent laws are violated in this process! Also, the general form of stable nonlinear dynamics is given in the continuous domain. If a digital architecture is desired (the preferred choice for implementation), any of the techniques to convert continuous domain equations to the discrete domain can be used. Or, we can form an analog architecture with the desired properties such as low sensitivity to element variations, for example, and transform it to the digital domain.

This rather simple approach can be used in a number of areas of nonlinear dynamics with great success. We will discuss the application to three problems in the general area of signal processing in section IV. First, we discuss the successful application of similar approaches in electrical engineering and other fields.

[3] This requires that the characteristics of the dynamic elements be confined to first- and third-quadrants.

3. PRECEDENCE

3.1. Constrained to Unconstrained Optimization Problem

In many engineering problems we are faced with the task of optimizing a multi-variable function subject to certain constraints in the range of values of the variables. Such a constrained optimization problem is more difficult than unconstrained optimization where the variables can take any values in the real domain. Thus, one common approach is to use transformations of variables to change the constrained optimization into an unconstrained optimization problem involving the new variables. As an example, to find the minimum of a function $f[x_1, x_2, x_3]$ of the three variables x_1, x_2, x_3 subject to the constraints, $0 \le x_1 < \infty$, $0 \le x_2 \le 1$ and $0 \le x_3 \le 1$, we can use respectively the transformations $x_1 = \hat{x}_1^2$, $x_2 = \tanh[\hat{x}_2^2]$ and $x_3 = \sin^2[\hat{x}_3^2]$. The minimum of $f[x_1, x_2, x_3]$ can then be found by unconstrained minimization of the new function $\hat{f}[\hat{x}_1, \hat{x}_2, \hat{x}_3]$.

3.2. Stability, Sensitivity & Filter Design Problems

There are a number of areas in filter design where such an approach has performed wonders. The first example is in the area of Active RC circuit design where the aim is to design micro-miniature integrated-circuit (IC) filters with characteristics that are possible with linear, passive R, L, C elements. Unfortunately, the inductive element L is not realizable using IC technology. However, using other elements such as operational amplifiers that can be realized in an IC form, one can build elements such as linear gyrators, that in turn could be combined with capacitors to simulate the behavior of inductors. A problem with such an approach is the network sensitivity. The changes in the elements' parameters (for example, the beta of the transistor or the open loop gain of the operational amplifier can vary considerably) can make the filter response change dramatically and render the designed filter useless. Another problem in simulating the behavior of inductors using operational amplifiers, is the need to have both terminals of the simulated inductors floating, as is the case of a low-pass or band-pass filter.

A clever solution to overcome these two problems simultaneously is based on the use of passive ladder and lattice architectures (whose responses are known to be least sensitive to variations in the element values) and a modified form of impedance scaling (by the complex frequency variable, 's') [5,6]. This transforms the R, L and C elements in the circuit respectively to C, R and FDNR (frequency dependent negative resistors) elements. All the FDNR elements in the transformed network have one grounded terminal and hence the new ladder network can be realized in an IC form. Thus, a simple but clever extension of the well known impedance scaling approach provides a way to eliminate the need for simulating ungrounded inductors in the ladder architecture and still preserve the low sensitivity property of the ladder architecture in realizing active-RC filters.

Other areas where such concepts have been used include design of 1D and 2D filters. In the case of 1D digital filters, the use of the bilinear transformation (which is known as a lossless transformation and hence preferred to convert analog equations to digital form) and low-sensitivity analog architectures have lead to low-sensitivity digital architectures known as *ladder digital filters* and *wave-digital filters [7]*.

We have more severe problem of fundamental nature in the case of 2D digital filters. Even the design of a 2D LTI filter requires that the zeros of a function of two complex variables be placed in a particular region. But they are not just few isolated zeros but a continuum of surfaces in four dimensions. This problem needs to be solved before we consider other practical issues such as low-sensitivity architecture etc. One solution that is now available [8, 9] involves the use of analog elements in the 2D domain, the circuits formed from such elements, and a double bilinear transformation to form prototype 2D digital filter equations. The use of only certain types of elements ensures that the obtained 2D digital filter equations are indeed stable for all range of values of the analog elements. This approach is identical to the building block approach for the design of stable nonlinear systems and the results obtained stands as an indication for the great potential of the building block approach

4. APPLICATION TO SIGNAL PROCESSING

In this section, we discuss the application of the new approach to three problems in signal processing. The first involves removing colored additive noise from a base-band digital, bipolar signal, a common problem in digital communication. When we use a LTI low-pass filter, we run into the classical trade-off between rise-time and bandwidth and hence the amount of noise energy that can be removed is limited. A time domain design technique leads to the well known LTI matched filter. Here, we can ask if nonlinear filters can be designed to do a better job. A solution is possible by combining the well-known concept of using preferred architectures from one domain to another. We start with a LTI low-pass ladder architecture designed perhaps based on frequency domain considerations. Next we make the reactive elements nonlinear and sculpture their I/O characteristics so as to open up the bandwidth (when bit transitions occurs) and reduce the bandwidth (when constant valued output is supposed to be present). The ladder architecture with inductors in the series (floating) arm and the capacitors in the parallel arm will allow us to easily choose their I/O characteristics. In our simulations, we used an architecture corresponding to a third-order Butterworth filter, with the characteristics leading to a bandwidth close to the one suggested by the LTI low-pass filter for steady bit value situations and a bandwidth 10 times that during bit transitions. This simple design leads to a nonlinear filter that does a better job of removing the in band noise. Complete details and the results of simulation using a 3rd-order nonlinear ladder filter at the conference will be presented at the conference. Further optimizations can make the filter more efficient.

This same problem, that of removing in band noise from a bipolar digital signal, can be configured as one in which the result has to correspond to one of two equilibrium states (+1 and -1) or attractive points. Thus, a second solution based on nonlinear network or dynamic neural network architecture can be designed for this job using the building block approach. For example, we can choose one reactive element, say a capacitor, to have a $v_c[q_c]$ Vs q_c characteristic that includes ± 1 as relaxation points and the origin as one where a local maxima of the stored energy takes place. We then assign q_c to the recovered bipolar bit stream. Next, we will incorporate the error between observed bit stream and the recovered one into the equation of another suitable element, say a nonlinear lossless gyrator and let the network oscillate. The combination of the two circuit equilibrium points (± 1) and the use of the error to guide move to the actual value will ensure that the proper bits are recovered. Once again, we will show the details and the results of some simulations at the conference.

The third application is in signal estimation. Kalman filter has become the standard tool in signal estimation. They correspond to time-varying filters where the Kalman gain the time-varying parameter and is modified based on some predetermined rule which does not include the error between the estimated signal and the observed signal. Thus, a Kalman filter can become unstable. Using the building block approach presented in this paper we can arrive at nonlinear architectures that include that error as well in modifying the Kalman filter such that the entire architecture is stable. That is one that corresponds to a nonlinear passive architecture either driven by the observed signal or one where the error signal is incorporated as a part of some element (a nonlinear gyrator, for example). Such a modification will guarantee the stability and contribute to better estimation and faster convergence. Further, we can arrive at low sensitive architectures that will lead to robustness in estimation. Again details will be provided at the conference.

Finally, as we mentioned before, our interest is in designing systems in the digital domain since it has become the de-facto standard. We use the analog domain to obtain suitable architectures and to solve complex stability issues. There is still another issue – what transformation to use to go from analog to digital domain. We noted that bilinear transformation is the preferred one in the LTI case because of its ability to carry over the important properties. However, BT has been kind of discarded in the nonlinear domain, as it will lead to implicit type of nonlinear difference equations, which will require an iterative procedure for each sampling instance. The advantages of bilinear transform combined with the improvements in the speed of VLSI circuits imply that the perceived problems are no longer a problem. We will address this issue in another paper.

5. SUMMARY

In this paper we are not claiming to advance any new jazzy, fundamental mathematical theory or some thing of that sort for nonlinear dynamical systems. Because most of the things that needs to be known and proven seems to have been taken care off and we have a good body theoretical knowledge from researchers in control theory, system theory, circuit theory, chaotic systems and so on. However, most of them are known and or can be understood only by a handful of people and the whole area is fragmented with super-specialization in a very narrow topic (special third-order chaotic nonlinear dynamics, for example). Thus, application of nonlinear dynamics is very sporadic and a very is an almost impossible task. That is where the contribution in this paper comes. We have presented a new approach for the design of nonlinear systems (signal processors as a special case) based on an entirely new and interesting paradigm. It is similar in spirits to the Neural nets paradigm (learn from existing systems) or the object oriented approach that has now become standard in software engineering and, in fact, takes it one step further as explained in the paper. We clearly show how nonlinear circuits (analog or digital) for various application areas can be designed with little or no effort using this approach. We have written a research manuscript / text-book that discusses the details of this new paradigm and show how it can easily be applied to a number of seemingly different areas such as signal processing, control theory, neural networks, chaotic systems and fuzzy systems. If you are the type who can understand some circuit theory limbo while researching control systems or can go beyond research in a single topic, let me know. I will be glad to send you an electronic copy of the manuscript free.

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